

Bichromatic Classifications using Strips

Jaegun Lee*

Chaeyoon Chung*

Hee-Kap Ahn†

Let P be a set of n points in the plane, each of which is colored either blue or red. In bichromatic separation, we aim to identify a *strip*, defined as a closed region bounded by two parallel lines of arbitrary orientation, whose placement separates the two colors as well as possible. We consider the following two variants: (1) locating a strip that contains no red points while maximizing the number of blue points within the strip (**MaxBlue**) and (2) locating a strip that contains all blue points while minimizing the number of red points within the strip (**MinRed**). Furthermore, we consider natural generalizations, t -**MaxBlue** and t -**MinRed**, respectively, in which up to t parallel strips are allowed for the separation for a fixed integer t . See Figure 1.

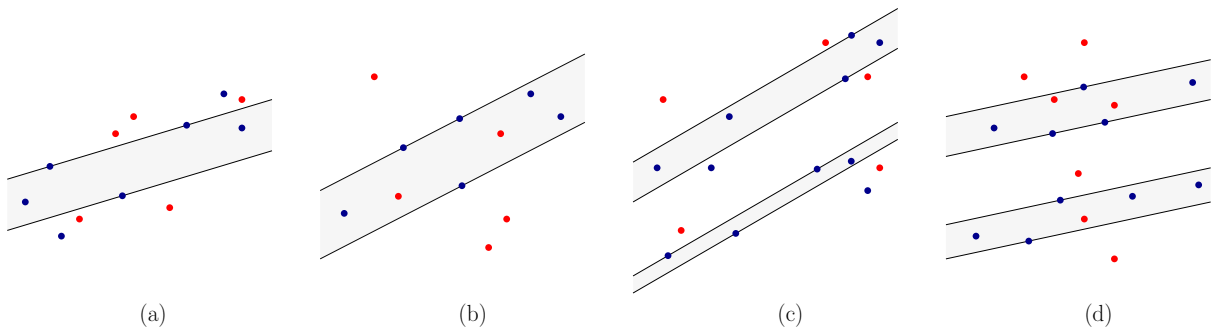


Figure 1: Optimal separation strips for (a) **MaxBlue**, (b) **MinRed**, (c) **2-MaxBlue**, (d) **2-MinRed**.

Our contribution. We give an $O(n^2)$ -time algorithm for **MaxBlue**, improving upon the previous best $O(n^2 \log n)$ -time result. We show that this running time is optimal under the standard 3SUM conjecture. The time complexities of our algorithm and the previous one depend only on n , the total number of input points. It is known that a perfect separation strip, wherein the two colors are distinctly separated without any errors, can be computed in $\Theta(n \log n)$ time if it exists, so there is a time gap when the number k^{opt} of blue points not within an optimal strip for **MaxBlue** is small.

Thus, it is preferable to have an algorithm whose running time depends on k^{opt} . We give an output-sensitive algorithm with running time $O(k^{\text{opt}} n \log n)$. We introduce an $O(n^2 \log n)$ -time algorithm for t -**MaxBlue**, for any value of t . For t -**MinRed**, we present an $O(n^2 \log n)$ -time algorithm for $t \geq 3$. When $t = 2$, we show that the problem is 3SUM-hard, and give an $O(n^2)$ -time algorithm.

*Department of Computer Science and Engineering, Pohang University of Science and Technology, Pohang, Korea, Email: {jagunlee, chaeyoon17}@postech.ac.kr

†Graduate School of Artificial Intelligence, Department of Computer Science and Engineering, Pohang University of Science and Technology, Pohang, Korea, Email: heekap@postech.ac.kr